

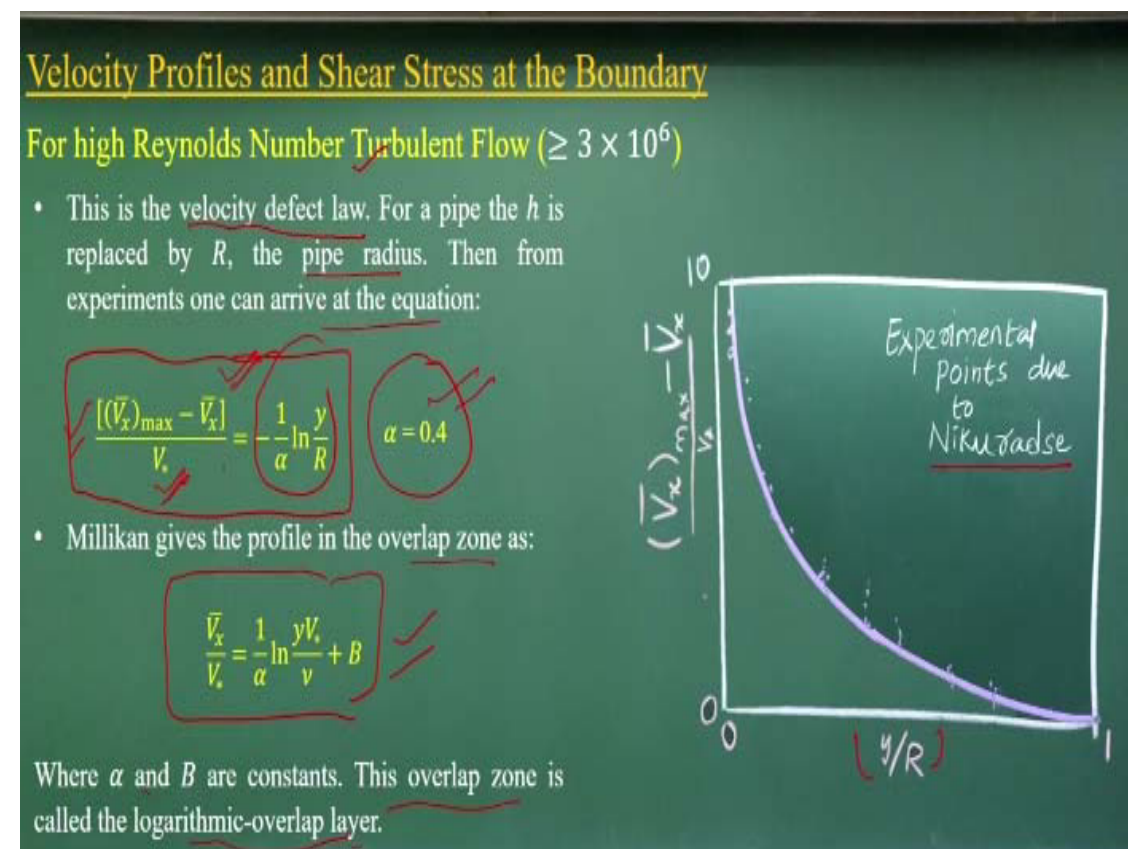
But if you go to the outer layers where we look it that a velocity defect concept, how far the velocity from average velocity, that the defect means how much deviations how much difference between that; if you look it that and looking,

$$[(\bar{V}_x)_{\max} - \bar{V}_x] = F(\tau_0, h, \rho, \nu)$$

We again we get the similar functions from the dimensional analysis between V and y and h ,

$$\frac{[(\bar{V}_x)_{\max} - \bar{V}_x]}{V_*} = F\left(\frac{y}{h}\right)$$

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Now if you look it that, if you put it high turbulence flow and the velocity the reasons is very this is called velocity defect law and h is replaced by the R the pipe radius then you can have this experimentally derived components and this α will represent a equal to the 0.4 and this is what the average velocity or time average velocity components and this is a special average velocity component how they are fluctuating with shear velocity.

$$\frac{[(\bar{V}_x)_{\max} - \bar{V}_x]}{V_*} = -\frac{1}{\alpha} \ln \frac{y}{R}$$

$$\alpha = 0.4$$

And more details if in a overlap zones you will have a this equation. So now if you look it from the experiment and the dimensional analysis using this Nikuradse experiment data set it was found what could be the α value okay which is here is 0.4 and for

the overlapped zones alpha, beta as a different value and here I am not talking much more. It is called the logarithmic overlap layers okay.

Millikan gives the profile in the overlap zone as

$$\frac{\bar{V}_x}{V_*} = \frac{1}{\alpha} \ln \frac{yV_*}{\nu} + B$$

In between these two regions we can locate how the velocity distributions is taking it. Now we coming back to the very simple examples okay. And that is what is in your text book is necessary to for. If you look it that, many of the times you have the pipes in series, pipes in parallel or three reservoir junction problems okay. Pipe in series is a very simple problems like electric circuits, okay. You can have a series of wires you have from point A to B, okay.

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Multiple-Path Pipe Flow

If systems contain two or more pipes, then certain basic rules are necessary to do calculations very smooth

- Pipes in series
- Pipes in parallel
- Three reservoir junction

Pipes in series:

Rule 1: The flow rate is the same in all pipes:

$$Q_1 = Q_2 = Q_3 = \text{constant}$$

$$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$$

Rule 2: Total head loss through the system equals the sum of the head loss in each pipe:

$$\Delta h_{in \rightarrow out} = \Delta h_1 + \Delta h_2 + \Delta h_3 + \dots$$

If you have definitely the discharge will be for a steady state conditions for steady flow conditions. So discharge at the Q_1 , Q_2 , Q_3 that should be equal because this is a steady state.

$$Q_1 = Q_2 = Q_3 = \text{constant}$$

$$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$$

But if you are having an energy losses between Δh_1 is energy losses for this regions.

This is

$$\Delta h_{in \rightarrow out} = \Delta h_1 + \Delta h_2 + \Delta h_3 + \dots$$

So this pipe in series a very simple problems, in which your flow is a constant okay.

But the total head loss is a sum of the head losses of individual pipe is connected in a series, very simple problems.

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Multiple-Path Pipe Flow

Pipes in series:

In terms of the friction and minor losses in each pipe:

$$\Delta h_{in \rightarrow out} = \frac{V_1^2}{2g} \left(\frac{f_1 L_1}{d_1} + \sum K_1 \right) + \frac{V_2^2}{2g} \left(\frac{f_2 L_2}{d_2} + \sum K_2 \right) + \frac{V_3^2}{2g} \left(\frac{f_3 L_3}{d_3} + \sum K_3 \right) + \dots$$

Since V_2 and V_3 are proportional to V_1

$$\Delta h_{in \rightarrow out} = \frac{V_1^2}{2g} (a_0 + a_1 f_1 + a_2 f_2 + a_3 f_3 + \dots)$$

But when you have a pipe in a series please remember it that you always should consider whether there is a minor losses. There is major losses which is the frictional losses component for the pipe 1. Also there will be a minor losses because the change of the diameter of the pipe okay there will be expansion or the contraction exit loss, entry losses all the loss component you have to look it which way the flow is happening it and you can quantify what could be the minor losses.

So this is the major loss plus minor loss. That is what is represented here for the pipe 1. This is the representing for the pipe 2, and this is the representing for pipe 3. So similarly, if are more number of pipes, you can have a just summations of the head loss, the head energy losses along the pipe from pipe 1, pipe 2, pipe 3. Most of the times if you look it because the same discharge is going through that, you will have the V 2 and V 3 which is a functions with a discharge is same.

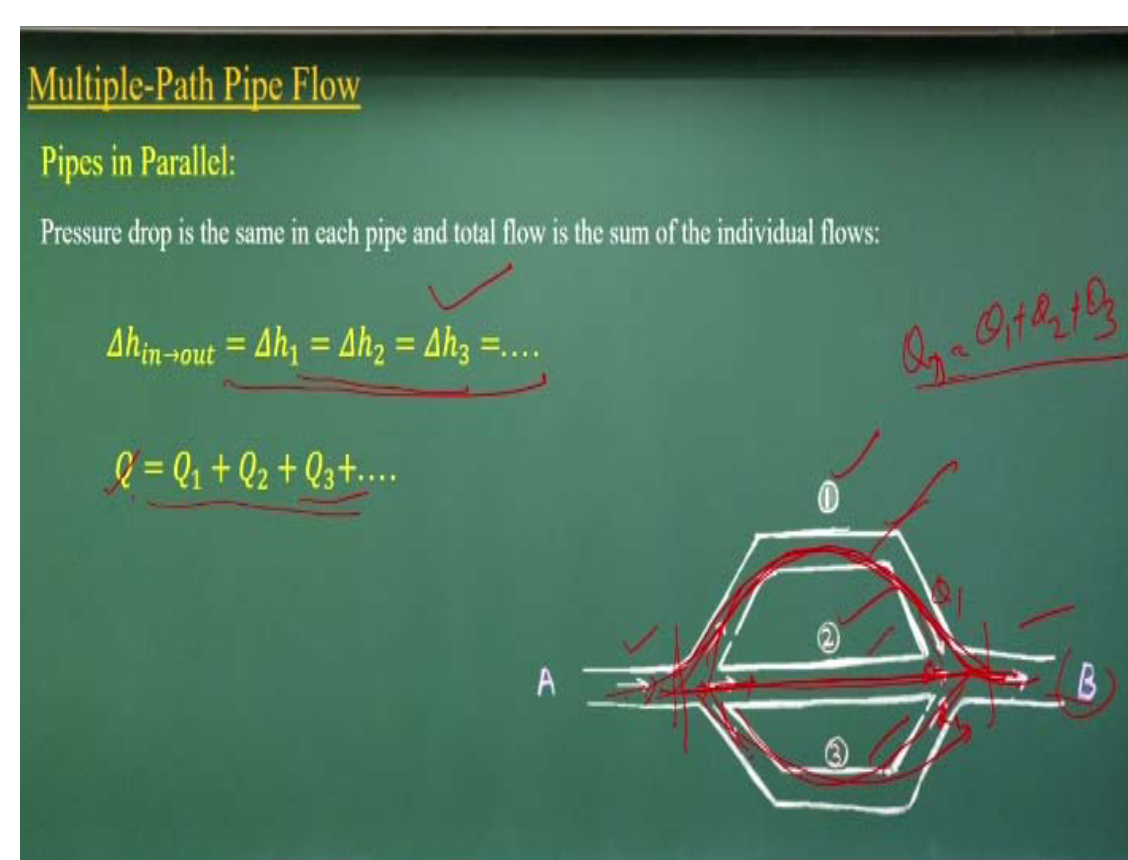
$$\Delta h_{in \rightarrow out} = \frac{V_1^2}{2g} \left(\frac{f_1 L_1}{d_1} + \sum K_1 \right) + \frac{V_2^2}{2g} \left(\frac{f_2 L_2}{d_2} + \sum K_2 \right) + \frac{V_3^2}{2g} \left(\frac{f_3 L_3}{d_3} + \sum K_3 \right) + \dots$$

It will have a proportionality with V_1 . So you can write a simple formulae like this okay So these are very shortcut way to do it, but I always encourage you please follow these the energy losses computations for major losses minor losses major losses minor losses and compute total.

Then find out what could be the energy loss is happening in a pipe in series conditions.
Now if you look it another very simple problems that pipes in parallel.

$$\Delta h_{in \rightarrow out} = \frac{V_1^2}{2g} (\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3 + \dots)$$

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When you have a pipe in a parallels, you can understand it see if I have the pipe in the parallels, there are three pipes are connected here. This is the A is entry point, B is exit point. From the this point to this point the total energy losses passing through this path A or path B or path C that should be equal. So energy losses should be equal, whether it follows a path A, path B, or path C, all the energy losses should be equal.

$$\Delta h_{in \rightarrow out} = \Delta h_1 = \Delta h_2 = \Delta h_3 = \dots$$

That is the conditions. And as the flow is coming and divide into three part, we can always write the Q what is coming it will be distributed into the three part in this figure. So as you have a branching out. As you are converging it, you can write

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

So sum of the discharge will give us the discharge what is passing through the A or B and the energy losses whether following the 1 path 2 path or 3 path should be equal. Then the energy at these two point will be come to a the same values. Otherwise it will not be possible to have a different energy losses. That cannot be happen it in a flow systems.

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Multiple-Path Pipe Flow

Three-Reservoir Junction:

All flows are considered positive toward the junction: $Q_1 + Q_2 + Q_3 + \dots = 0$

The pressure must change through each pipe so as to give the same static pressure P_j at the junction: Let the HGL at the junction have the elevation

$$h_j = Z_j + \frac{P_j}{\rho g}$$

Where is in gage pressure for simplicity

Head loss through each, assuming $p_1 = p_2 = p_3 = 0$ (gage)

$$\Delta h_1 = \frac{V_1^2 f_1 L_1}{2g d_1} = z_1 - h_j$$

$$\Delta h_2 = \frac{V_2^2 f_2 L_2}{2g d_2} = z_2 - h_j$$

$$\Delta h_3 = \frac{V_3^2 f_3 L_3}{2g d_3} = z_3 - h_j$$

Now if you look at the three reservoir junction problems which many of the time it is given that you have a multiple reservoirs okay. You may have the multiple water tanks are there and connected to the pipe flow systems and you have a junction where you have three are connected here. Out of these three flow one could be also outflow. We do not know that which direction it will be, outflow it.

But we can say that some of the Q discharge should be equal to zero at this point.

$$Q_1 + Q_2 + Q_3 + \dots = 0$$

There is no outlet, okay. So at the junction point, the continuity equations are mass conservations at these equations should be equal to zero. But another one what you need to compute it how much of energy or the head is what is the elevations of high gradient line.

$$h_j = Z_j + \frac{P_j}{\rho g}$$

These hydraulic gradient is supposed to be equal to yours the change of we have this gradient line at this point like this. And I know this hydraulic gradient lines at these points. If I looking that how much of head loss is happening because the pressure head is equal to zero here, the velocity head equal to zero here.

assuming $p_1 = p_2 = p_3 = 0$ (gage)

So this is energy gradient line, also the hydraulic gradient line. So that way if I look at these the line of hydraulic gradient line, not the energy gradient line, line of hydraulic

gradient line because I am not looking the velocity point. That should be equal at this point after having the energy losses.

$$\Delta h_1 = \frac{V_1^2}{2g} \frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g} \frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g} \frac{f_3 L_3}{d_3} = z_3 - h_J$$

So how much of energy losses will happen in such a way that we will have a hydraulic gradient at the junctions will be this ones. That is what we apply with a loss and compute what is the head losses should be there in this direction in this direction and in this direction, okay. And based on that we can compute that what will be the flow contributions Q 1 or Q 2 and Q 3 directions.

So you have a three equations as well as this equations help us to find out the flow directions and the head losses of a three reservoir junctions problems okay. This quite interesting problems but try to understand that we are computing the hydraulic gradient line and what is the hydraulic gradient locations of at the junctions and based on that, we are just equating the energy losses to compute that part.

That is what the basic (()) (36:49). Now come back to a example one which is a GATE 2014 question paper which looks like very lengthy but is a very simple problems okay.

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Example 1

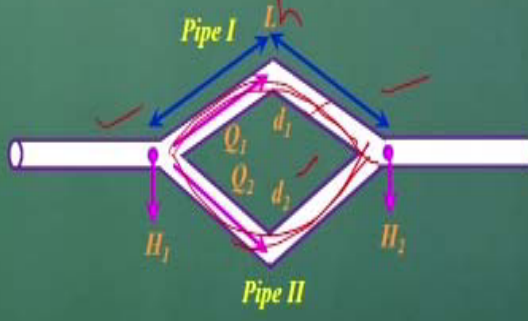
An incompressible fluid is flowing at steady rate in a horizontal pipe. From a section, the pipe divided into two horizontal parallel pipes (of diameter d_1 , d_2 and $d_1=4d_2$) that run for a distance of L each and then again join back to a pipe of the original size. For both the parallel pipes, assume the head losses due to friction only and the Darcy-Weisbach friction factor to be same. The velocity ratio between bigger and smaller branched pipe is? (GATE 2014 SET I)

Flow classification:

- One dimensional
- Steady flow
- Incompressible flow
- Homogeneous fluid

Assumptions:

- Neglected minor losses



An incompressible fluid flowing through a steady rate in a horizontal pipe, okay. From a section, pipe is divided into two horizontal pipes as given the d_1 and d_2 okay and d_1 is four times of d_2 run for a distance l each other. Then again join back to pipe of the original size okay. We will just sketch the problem, okay? Many of the times please sketch the problems then you can understand it.

For both the parallel pipe assume the head loss due to the frictions only. Darcy-Weisbach friction factor to be same. So f is a constant, f is constant. What could be the velocity ratio between bigger and smaller branched pipe, okay. If you look at this problems, as we solved the earlier we follow this basic concept of one dimensional steady flow incompressible this neglecting the minor losses only the friction losses to be consider it.

As is the parallel pipe flow you know these head losses of this path pipe 1 and pipe 2 should be equal. This is very simple; f is a constant and the head losses between either the pipe 1 or pipe 2 should be equal. Based on that you can compute what could be the d_1 , d_2 . Ratio is known to us. We have to compute the velocity ratio.

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Example 1

Case I: Flow through pipe I
head losses between section 1 and 2 for pipe I

$$H_1 - H_2 = \frac{fLV_1^2}{2gd_1}$$

Case II: Flow through pipe II
head losses between section 1 and 2 for pipe II

$$H_1 - H_2 = \frac{fLV_2^2}{2gd_2}$$

$$\frac{fLV_1^2}{2gd_1} = \frac{fLV_2^2}{2gd_2}$$

$$\frac{V_1^2}{4d_2} = \frac{V_2^2}{d_2}$$

$$\frac{V_1}{V_2} = 2$$

Now if you look at this way, is very simple things, that you know it head losses between section 1 and 2 for this. These two if I equate it I will get it the ratio of the velocity. That is what is the problem, okay. Even if I do not need continuity equations to show it. Only with alpha the head losses between section a and 2 being a parallel pipe that should be equal and that is what if you equate it I will get the ratio between the velocity V 1 and V 2 okay. That is the point what we derived.

Case I: Flow through pipe I

head losses between section 1 and 2 for pipe I

$$H_1 - H_2 = \frac{fLV_1^2}{2gd_1}$$

Case II: Flow through pipe II

head losses between section 1 and 2 for pipe II

$$H_1 - H_2 = \frac{fLV_2^2}{2gd_2}$$

$$\frac{fLV_1^2}{2gd_1} = \frac{fLV_2^2}{2gd_2}$$

$$\frac{V_1^2}{4d_2} = \frac{V_2^2}{d_2}$$

$$\frac{V_1}{V_2} = 2$$

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Example 2

The minimum gradient is provided such that it accommodate the friction losses.

$h = h_f$

$A \times V = Q$

$\frac{\pi d^2}{4} \times 0.75 = 0.21$ $d = 0.597 \text{ m}$

$h_f = \frac{fLV^2}{2gd} = \frac{0.01 \times 100 \times (0.75)^2}{2 \times 9.81 \times 0.597} = 4.8 \text{ cm}$

Minimum gradient $\frac{h_f}{L} = \frac{4.8}{100} = 0.048$

Data Given:

$Q = 0.21 \text{ m}^3/\text{s}$
 $V = 0.75 \text{ m/s}$
 $f = 0.01$

Another problems which is GATE 2014 questions also. A straight 100 meter long raw water, okay this only waters gravity main is carry the water from intake to a jack well okay of a water treatment plant okay. So basically there is a intake, to take the water from the rivers or ground waters and go to a well which is called the jack well where basic screening and all things turn it and then go for the water treatment plant.

Required flow through the water main is given to us. The allowable velocity is given to us 0.75 meter per second; f is given g is given to us. The minimum gradient what could be the minimum gradient to be given to the main so that it require amount of flow without any difficulties okay. We just sketch the problems. The problem is the 100 meters, there is a intake. This is the jack, there the Q is given to us.

Data Given:

$$Q = 0.21 \text{ m}^3/\text{s}$$

$$V = 0.75 \text{ m/s}$$

$$f = 0.01$$

The Q is given to us and also it is given the allowable velocity. If this is the maximum velocity can go through this pipe. What is looking it what could be the minimum gradient to given to this pipe so that the required amount of water can go to the jack well from the intake well. The problems if you look it, it looks very difficult but it is not that.

What is looking it that how much of energy losses is happening it when flow is going from A to B because of frictional losses, how much of energy loss is happening it. Whether we can put a gradient such a way that it can make a flow maintain it. That means you compute the energy losses multiply it with the length. That what will gives us totally energy loss what is happening it.

And if I know the total energy losses, then I can know it what could be the gradient or the potential energy head I have to provide it to have this flow system. Again I am to repeat these things to tell it that the problem is frame it in different way but try to understand it. What is looking it that what could be the minimum gradient to be given to these flow, so that water can flow from this point intake to the jack well.

$$h = h_f$$

That means, when flow is going through this pipe there will be a frictional losses. The energy losses due to the pipe frictions. Those frictions should be energy losses should be overtaken by the potential energy gradient what we have to keep it. That is the problem. So potential energy gradient to be provided with respect to the energy losses what is going to happen it.

If you do not provide minimum gradient, then water will not flow through that. That is the very basic concept here. The minimum gradient is to provide such as that it accommodates the frictional losses, okay. That is the trick of the problem. Since the velocity is given, the Q is given, we know what could be the diameters using the continuity equations, very simple things.

$$A \times V = Q$$

$$\frac{\pi d^2}{4} \times 0.75 = 0.21$$

$$d = 0.597 \text{ m}$$

$$h_f = \frac{fLV^2}{2gd} = \frac{0.01 \times 100 \times (0.75)^2}{2 \times 9.81 \times 0.597} = 4.8 \text{ cm}$$

Minimum gradient

$$\frac{h_f}{L} = \frac{4.8}{100} = 0.048$$

And you also know it what could be the energy losses in terms of meter. Just substituting these values okay you can get it, what could be the energy losses. Then the

minimum gradient what you were looking. This the gradient is necessary to have flow from intake to jack well.

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Example 3

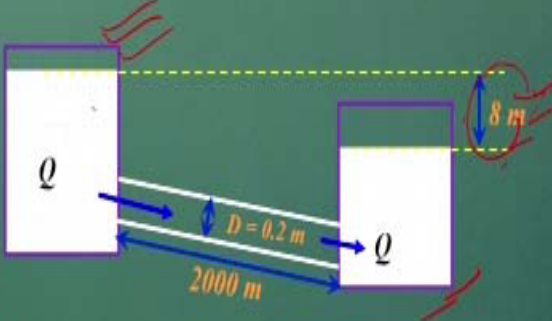
A 2 km long pipe of 0.2 m diameter connects two reservoirs. The different between water levels in the reservoirs is 8m. The Darcy- Weisbach friction factor of the pipe is 0.04. Accounting for friction, entry and exit losses. The average velocity of the flow is

Flow classification:

- One dimensional
- Steady flow
- Turbulent
- Incompressible flow
- Homogeneous fluid
- Friction flow

Assumptions:

- Flow will take place due to total head of 8m
- Difference of elevations between water surface in the reservoirs is the sum of major losses (friction) and minor losses (entry, exit, contraction, expansion, valves, bends, elbows etc.)



Let us start this third examples which is gives that 2 kilometer long pipe with a diameter of 0.2 meter diameter connects two reservoirs as given in the figures. The difference between the water levels in the reservoir is 8 meters. That means the energy losses between two reservoir is 8 meters. Darcy-Weisbach friction factor is given to us which is 0.04.

Accounting friction, entry and the exit losses we have to estimate the average velocity of the flow. So that way we can say it the flow will take place with having total energy losses of 8 meters and difference between these energy losses will account for major losses, which is a friction losses. The minor losses here is the entry and the exit losses. That is what the additional things what will consider it, but other losses we will not consider this case as given in the problems.